Example 6

Suppose x, y, and z are three quantities such that $4x + 4y + 3z = 30$ and $3x + 3y + 2z = 10$. What is the average of x, y, and z?

F. $6\frac{2}{3}$

G. $7\frac{1}{2}$

H. 8

J. 10

K. $13\frac{1}{3}$

$4x+4y+3z=30$

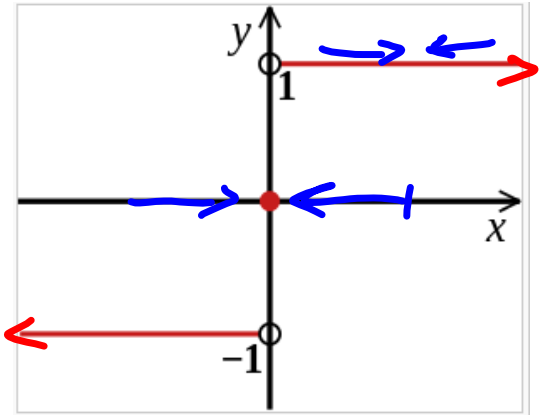
$-(3x+3y+2z=10)$

$\frac{x+y+z=20}{3}$ $\frac{20}{3}$

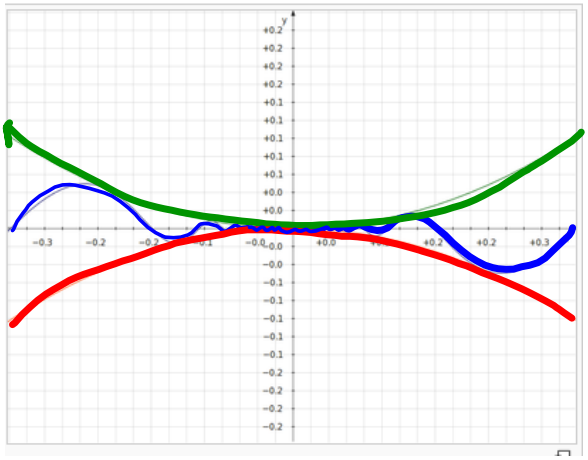
$\frac{x+y+z}{3}$

The Signum Function sgn

$$sgn(x) = \begin{cases} -1 & x < 0 \\ 0 & x = 0 \\ 1 & x > 0 \end{cases}$$



The Squeeze Thm



f(x)

h(x)

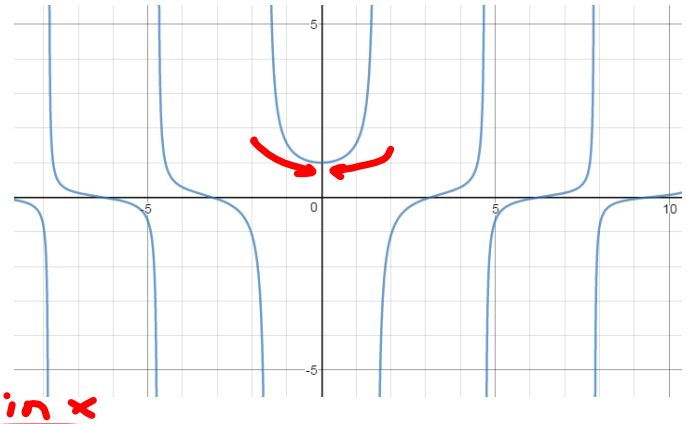
g(x)

$\lim_{x \to 0} h(x)$

$\lim_{x \to 0} g(x) \leq \lim_{x \to 0} h(x) \leq \lim_{x \to 0} f(x)$

$$\lim_{x \to 0} \frac{\sin x}{x} = 1 \qquad \lim_{x \to 0} \frac{1 - \cos x}{x} = 0$$

$$\lim_{x\to 0}\frac{\tan x}{x}$$



$\lim_{x\to 0} \frac{1}{x} \tan x$

$\lim_{x\to 0} \frac{1}{x} \frac{\sin x}{\cos x} = \frac{\sin x}{x \cos x}$

$\lim_{x\to 0} \frac{\sin x}{x} \cdot \frac{1}{\cos x}$

$1 \cdot 1$

$\boxed{1}$

$$\lim_{x\to 0}\frac{\sin 4x}{x}$$

$\lim_{x\to 0}$ $\frac{4}{4}\frac{\sin 4x}{x} = \frac{4\sin 4x}{4x}$

$4\lim_{x\to 0}\frac{\sin 4x}{4x} : \frac{\sin u}{u}$ $u = 4x$

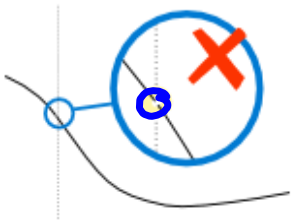
$4 \cdot 1$

4

Continuity

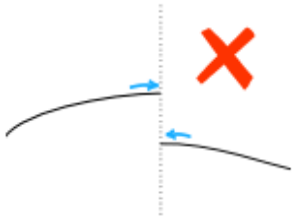
Continuous

- at specific points
- over an intrval



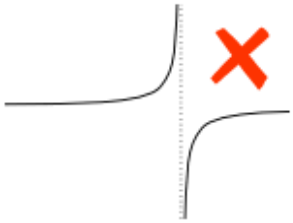
Not Continuous
(hole)

The limit is not defined at x=c

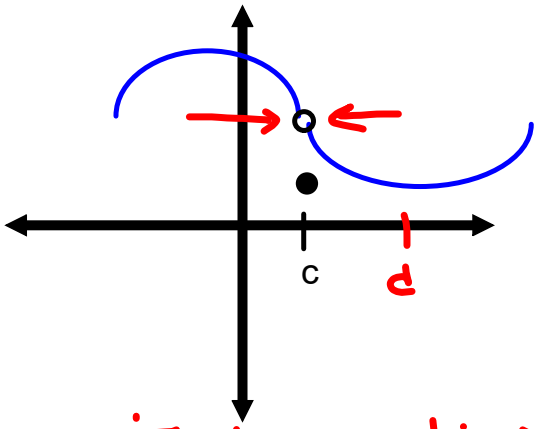


Not Continuous
(jump)

The limit of f(x) does not exist



Not Continuous
(vertical asymptote)



The limit exists but does not equal f(c)

= not continuous at c

is it continuous

(c, ∞) yes

[-1, d) no

All 3 conditions must be met for a function to be continuous at a point.

1. f(c) is defined
2. $\lim_{x \to c} f(x)$ exists
3. $\lim_{x \to c} f(x) = f(c)$

To be continuous on an open interval... $(-\infty, \infty)$

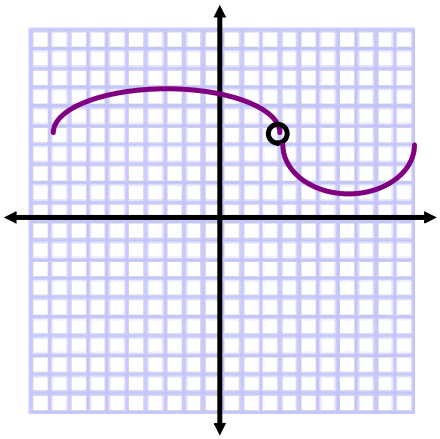
A function must be continuous at every point on the interval.

If a function is continuous on the interval $(-\infty, \infty)$, then it is **everywhere continuous.**

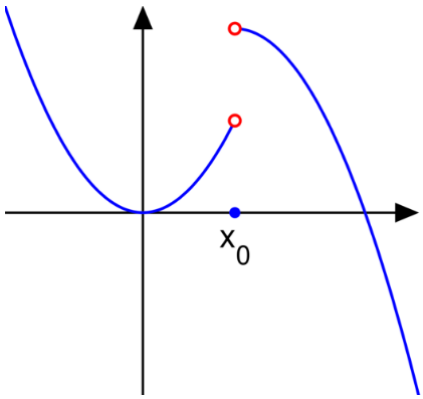
$y=x$ $y=x^2$ $y=3.5$ $y=|x^3|$

discontinuity

Removable



Nonremovable



Continuity of ....

$$f(x) = \frac{1}{x}$$

not continuous
b/c V.A.
not removable

$$h(x) = \begin{cases} x+1 & x \leq 0 \\ x^2+1 & x > 0 \end{cases}$$

0+1 = 1 (0,1]
0+1 = 1 (0,1)

yes cont.

$$f(x) = \frac{x^2-1}{x-1}$$

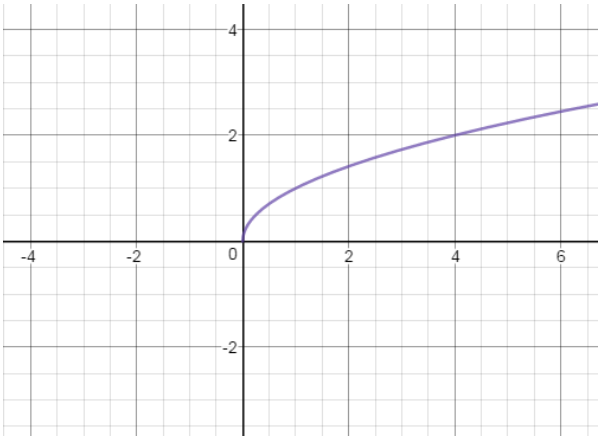
not cont
b/c hole
removable

$$y = \sin x$$

yes

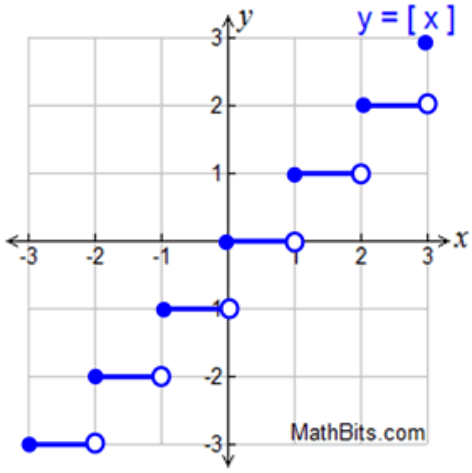
One sided limits

$$\lim_{x \to 0^+} \sqrt{x} = 0$$



$$\lim_{x \to 2^+} \sqrt{4 - x^2}$$

$[\![x]\!]$



$$\lim_{x \to 0^-} [\![x]\!] = -1$$

$$\lim_{x \to 0^+} [\![x]\!] = 0$$